Lecture 8

Wednesday, September 21, 2016   9:03 AM

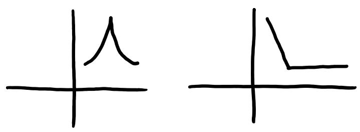
$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

Domain of $f'(x)$ consists of all points in the domain of f where the above limit exists.

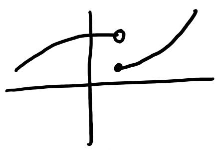
FROM LAST CLASS

HOW CAN A FUNCTION FAIL TO BE DIFFERENTIABLE :

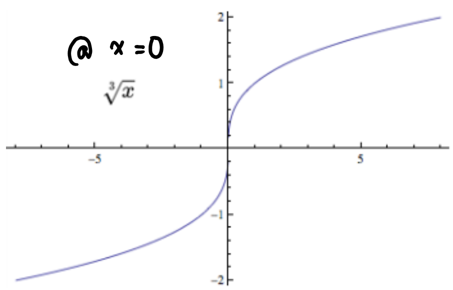
1) Kink or corner



2) DISCONTINUITY



3) Vertical tangent line



$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} \quad \text{DEF}$$

FUNCTION	DERIVATIVE
• c	0
• $x^n, n\in\mathbb{R}$	nx^{n-1}
• $f+g$	$f'+g'$
• $f-g$	$f'-g'$
• cf	cf'

$f(x) = c$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{c-c}{h} = \lim_{h\to 0} \frac{0}{h} = 0$$

$f(x) = x^3$

$f'(x) = 3x^{3-1} = 3x^2$

$$\frac{d}{dx}[cf(x)] = c\frac{d}{dx}[f(x)]$$

Ex $f(x) = 4x^3 + 6x^2 + 9$

Find $\frac{d}{dx}[f(x)]$

$$= \frac{d}{dx}\left[4x^3 + 6x^2 + 9\right]$$

$$= \frac{d}{dx}[4x^3] + \frac{d}{dx}[6x^2] + \frac{d}{dx}[9]$$

$$= 4\frac{d}{dx}[x^3] + 6\frac{d}{dx}[x^2] + 0$$

$$= 4.3x^2 + 6.2x^1 + 0 = 12x^2 + 12x$$

FUNCTION	DERIVATIVES	
• $\sin x$	$\cos x$ *	
• $\cos x$	$-\sin x$ *	
• fg	$f'g + fg'$	PRODUCT RULE
• $\frac{f}{g}$	$\frac{gf' - g'f}{g^2}$	QUOTIENT RULE

Ex $y = \tan x$, find $\frac{dy}{dx} = \sec^2 x$

$$\frac{d}{dx}(\tan x) = \frac{d}{dx}\left(\frac{\sin x}{\cos x}\right) = \frac{\cos x.\cos x - (-\sin x).\sin x}{\cos^2 x}$$

$$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x} = \sec^2 x$$

DIY Similarly find derivatives of $\operatorname{cosec} x$, $\sec x$, $\cot x$

Ex Find $f'(x)$ where $f(x) = \underbrace{x^3}\,\underbrace{\sin x}$

$f'(x) = x^3 \cdot \cos x + 3x^2 \cdot \sin x$

Ex $F(x) = \dfrac{3x^2 + 6x + 8}{x}$, $F'(x) = ?$

$= 3x^1 + 6 + 8x^{-1}$

$F'(x) = 3 \cdot 1 x^{1-1} + 0 + 8 \cdot (-1) x^{-1-1}$

$= 3 - 8x^{-2} = 3 - \dfrac{8}{x^2}$

FUNCTION	DERIVATIVE
• e^x	• e^x
• a^x $(a>0,\ a \neq 1)$	• $a^x \ln a$

Ex $F(x) = \underbrace{3^{-x}}\cdot\underbrace{\tan x}$, $F'(x) = ?$

$= \dfrac{1}{3^x} \cdot \tan x$

$= \dfrac{\tan x}{3^x}$ $\qquad \left(\dfrac{f}{g}\right)' = \dfrac{gf' - g'f}{g^2}$

$F'(x) = \dfrac{3^x \cdot \sec^2 x - 3^x \ln 3 \cdot \tan x}{(3^x)^2}$

$= \dfrac{\cancel{3^x}\left[\sec^2 x - \ln 3 \tan x\right]}{(3^x)^{\cancel{2}}}$

$= \dfrac{\sec^2 x - \ln 3 \tan x}{3^x}$

Ex Find the equation of the Tangent line to the curve $y = \dfrac{\sqrt{x}}{1+x^2}$ at pt $\left(1, \frac{1}{2}\right)$

$\frac{dy}{dx} = \dfrac{(1+x^2)\frac{d}{dx}(\sqrt{x}) - \frac{d}{dx}(1+x^2)\sqrt{x}}{\ldots}$

$$\frac{dy}{dx} = \frac{(1+x^2)\frac{d}{dx}(\sqrt{x}) - \frac{d}{dx}(1+x^2)\sqrt{x}}{(1+x^2)^2}$$

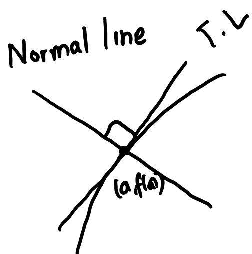
$$= \frac{\left((1+x^2)\cdot\frac{1}{2\sqrt{x}} - 2x\cdot\sqrt{x}\right)}{(1+x^2)^2} \quad \frac{2\sqrt{x}}{2\sqrt{x}}$$

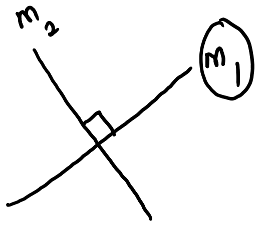
$$= \frac{(1+x^2) - 4x^2}{2\sqrt{x}\,(1+x^2)^2} = \frac{1-3x^2}{2\sqrt{x}(1+x^2)^2}$$

$$m = \left.\frac{dy}{dx}\right|_{(1,\frac{1}{2})} = \frac{1-3(1)^2}{2\sqrt{1}(1+1^2)^2} = -\frac{1}{4}$$

Eqn of TL $\quad y - y_1 = m(x - x_1)$

$$y - \frac{1}{2} = -\frac{1}{4}(x-1)$$





$m_1 = -\frac{1}{4}$

$$m_1 . m_2 = -1$$

$$m_2 = -\frac{1}{m_1} = 4$$

$$y = -\frac{1}{4}x + \frac{1}{4} + \frac{1}{2}$$

Eqn of normal line $\quad y - \frac{1}{2} = 4(x-1)$

$$y = 4x - 4 + \frac{1}{2}$$

Higher Derivatives

$f \cong$ diff func $\longrightarrow f' \equiv$ func

$(f')' = f''$ second derivative of f

$y - y_1 = m(x - x_1)$

$''$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right)$$

$m, (x_1, y_1)$

$$\boxed{y = mx + b}$$

$$f^{(3)}(x) = \frac{d^3y}{dx^3} \quad \ldots\ldots \quad f^{(n)}(x) = \frac{d^ny}{dx^n}$$

$y_1 = mx_1 + b$

$b = y_1 - mx_1$

$y = mx + y_1 - mx_1$

$f(x) = 14x^{15} + x^{14}$

$y - y_1 = mx - mx_1$